# CC - Matrices

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| **When is matrix multiplication allowed?** | The column number of the left matrix = row number right matrix.    You say they’re **conformable to multiplication**. |
| **What is the determinant of a 2x2 matrix?** |  |
| **What is det(AB) equal to?** | det(**A**) x det(**B**) |
| **How do you find the inverse of a 2x2 matrix?** | If…    Then… |
| **Describe the order of composite transformations** | Described as A followed by B followed by C. |
| **What does the determinant represent?** | The change in area/volume of a shape under the transformation. |
| **When is orientation preserved under a transformation? What does it mean when it isn’t?** | When…    Less than 0 means some reflection is involved in the transformation. |
| **What is the determinant of a 3x3 matrix?** | You can also used any other row / column yet have to follow these +’s and -’s. |
| **When is a matrix singular? When is this useful?** | When…    Can be used to see whether a system of equations has a solution. |
| **How do you find the inverse of a 3x3 matrix?** | 1. Calculate the determinant. 2. Find the minor of each value with alternating +’s and -’s. 3. Transpose this matrix of cofactors. 4. Divide by the determinant.     *Cofactors are denoted by capital letters:*    *Hence why | matrix | = a1A1 + b1B1 + c1C1 yet B1 is negative.* |
| **When are equations inconsistent in ‘systems of equations’?** | When they don’t have a unique point of intersection. |
| **Give the 3 cases where systems of equations have no unique point and the conditions of each** | 1. When two of the planes are parallel:    Easy to check if any 2 are parallel.  2. Form a triangular prism:    Eliminating 1 variable will show they are inconsistent.  3. They form a sheaf:    Eliminating 1 variable will show they are consistent. |
| **What are invariant points?** | Points which remain unchanged under a transformation. |
| **What is a line of invariant points?** | A whole line of points, each which remains unchanged under a transformation. |
| **Describe what happens to invariant lines under a transformation** | When every point on the line is mapped to **ANOTHER** point on the same line. |
| **How can you find invariant points / lines of invariant points OR invariant lines?** | Invariant points / lines of points:    Invariant lines: |
| **What is (AB)T equal to?** | (**AB**)T = **BTAT** |
| **What is det(A-1) equal to? How can this be proved?** | Can be proved using the fact det(**AA-1**) = det(**I**) |
| **What is (AB)-1 equal to? Why?** | **B-1A-1** since… |
| **What is det(MT) equal to?** | det(**M**) |
| **What row operations can be carried out on a matrix determinant? What effect can these have?** | * No effect on determinant value:   + Adding or subtracting any multiple of a row to another row or column to another column. * Changes sign of determinant value:   + Swapping two rows or two columns. * Changes determinant value by scalar:   + Multiplying/dividing a row or column by a scalar will multiply/divide (respectively) the determinant by that same scalar.   *Thus to ensure the value isn’t changed, you will have to add a minus if swapping or multiply when dividing or dividing when multiplying.* |
| **How can you that something is a factor of a determinant?** | To show (x - y) is a factor of the determinant, we substitute x = y into it and show that the determinant is now equal to zero.    Becomes…    Which becomes (when col 1 - col 2):    *Hence, we’ve also shown when 2 columns or rows are equal, the determinant is always 0.* |
| **What is an eigenvector and an eigenvalue?** | * Eigenvector - a vector whose direction is maintained under a transformation. * Eigenvalue - the value by which the eigenvector is scaled under that transformation.   This satisfies the equation **Ax** = λ**x**. |
| **What is the characteristic equation for an eigenvector? And how is it derived?** |  |
| **How is a matrix diagonalised? What is important here?** | *The equation above comes from* ***P-1MP*** *=* ***D****. This works because we first apply transformation P which turns our basis vectors into the eigenvectors. Now these eigen basis vectors are scaled using the transformation* ***M*** *(which is responsible for the eigenvectors in* ***P****). Their direction doesn’t change. Now they are transformed back into our basis vectors of [(1, 0) (0,1)]. This is identical to purely having the eigenvalues (the values by which they’re scaled) as the basis vectors.* |
| **How is diagonalisation of a matrix useful? (with derivation)** |  |
| **How do eigenvectors relate to lines of invariant points?** | From **Tx** = λ**x** …  If a transformation given by a matrix **T** has an eigenvalue of 1 then the corresponding eigenvectors determine the direction of a line of invariant points through the origin. |
| **How is a shear parallel to one axis represented?** | Left is a shear parallel to the x-axis. Right is a shear parallel to the y-axis. |